On the blank piece of paper, please write

1) Your name at the top, and Ch. 7 & 8

2) Title the paper: **PROBABILITY RULES CHEAT SHEET**

Example:

John Smith Ch. 7 & 8

PROBABILITY RULES CHEAT SHEET

Introductory Vocab (use the book, chapter 7, to get the definitions)

Experimental Probability- the probability that ACTUALLY happened in the experiment.

Ex: If I toss a coin 30 times, and get 12 heads, what's the **experimental** prob. of getting heads?

Theoretical Probability- the probability that SHOULD have happened in the experiment.

Ex: Using the same coin tossing situation above, what's the **theoretical** prob. of getting heads?

Probability Models- 2 parts:

* A list of all possible outcomes.
* The probability of each of those outcomes.

EXAMPLE: M&Ms

Color	Prob.
Brown	30%
Red	20%
Yellow	20%
Green	10%
Orange	10%
Blue	?

Sample Space- All possible outcomes

Example: What's the sample space for the Tony Gwyn experiment?

What about when I roll 2 dice and look at the sum?

What about the "Let's make a Deal" experiment?

Event- An outcome, or set of outcomes.

Ex: When rolling one die, events could be:

Evens, a 5, less than 4, etc.

Probability Notation:

- A, B, C, etc. =
- P(A) =
- S =
- A^c =

Basic Probability Rules

1) _____ **≤ P(A) ≤** _____

2) **P(S) =**

3) **$P(A^c)$ =**

Write these 3 probability rules on your cheat sheet!

Example 1: If the probability of hitting a homerun is 30%, what's the probability of not hitting a homerun?

$P(H) =$ $P(H^c) =$

Example 2: If there are only 8 different blood types, fill in the chart below:

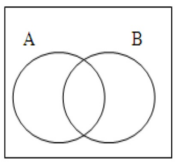
Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	?	0.07	0.1	0.11

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

Basic Symbols:

Intersection:

- Meaning:
- Symbol:
- Example 1:



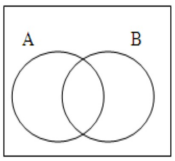
$P(A) = 0.50$ $P(B) = 0.70$ $P(A \cap B) = 0.30$

- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  $A \cap B$ = A and B = { }

Union:

- Meaning:
- Symbol:
- Example 1:



$P(A) = 0.50$ $P(B) = 0.70$ $P(A \cap B) = 0.30$

- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  A U B = A or B = { }

TRY THESE:

Sample Space = {1, 2, 3, 4, ... 22, 23, 24, 25}
A = {1, 3, 6, 7, 8, 10, 11, 13, 14, 15} B = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21}
C = {2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24}

1) What is $A \cap B$?
   {3, 7, 11, 13, 15}
2) What is A U B?
   {1,3,5,6,7,8,9,10,11,13,14,15,17,19,21}
3) What is B^c?
   {1,2,4,6,8,10,12,14,16,18,20,22,23,24,25}
4) What is $C \cap B^c$?
   {2,4,6,8,10,12,14,16,20,22,24}
9) What is $P(A \cap B)$?
   5/25 = 20%
10) What is $P(A \cup B)$?
   15/25
11) What is $P(B^c)$?
   15/25
12) What is $P(C \cap B^c)$?
   11/25

TRY THESE:

Sample Space = {1, 2, 3, 4, ... 22, 23, 24, 25}
A = {1, 3, 6, 7, 8, 10, 11, 13, 14, 15} B = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21}
C = {2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24}

5) What is $A \cap B \cap C$?
   nothing! empty set $\{\} \emptyset \{\}$
6) What is A U B U C?
   {1 -- 17, 19, 20, 21, 22, 24}
7) What is $A \cap C$?
   {6, 8, 10} $\{\}$
8) What is C^c?
   {1,3,5,7,9,11,13,15,17,18,19,21,23,25}
13) What is $P(A \cap B \cap C)$?
   0%
14) What is $P(A \cup B \cup C)$?
   22/25
15) What is P(A)?
   10/25
16) What is P(B)?
   10/25

17) Draw a Venn Diagram for the following. Then answer the questions below.
P(R) = 0.26
P(X) = 0.41
P(R ∩ X) = 0.12

(a) P(R U X) =

(b) P(R^c) =

(c) P(X^c) =

(d) P(R ∩ X^c) =

(e) P(X ∩ R^c) =

(f) P(R U X)^c =

BASIC PROBABILITY RULES: Unions & Intersections

Example #1: We are picking one card out of a standard 52-card deck.

What is the probability of picking a diamond? $P(D) = \frac{13}{52}$

What is the probability of picking a 3? $P(3) = \frac{4}{52}$

What is the probability of picking a diamond or a 3? $P(D \cup 3) = \frac{13+4-1}{52} = \frac{16}{52}$

What is the probability of picking a black card? $P(B) = \frac{26}{52}$

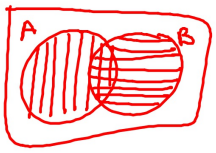
What is the probability of picking a Jack? $P(J) = \frac{4}{52}$

What is the probability of picking a black card or a Jack? $P(B \cup J) = \frac{26+4-2}{52} = \frac{28}{52}$

So… to find the probability of event A OR event B we….

add A and B, but subtract overlap

P(A or B) = P(A U B) = $P(A)+P(B)-P(A \cap B)$



A U B

EXAMPLE: There are only 8 different blood types, given below

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	0.06	0.07	0.1	0.11

What is the probability of being either Type A+ or B-? $P(A+ \cup B-) = 0.33$

What is the probability of being either Type O- or O+? $P(O- \cup O+) = 0.21$

What is the probability of being either Type AB+ or A+? $P(AB+ \cup A+) = 0.22$

What was different about this example?

no overlaps!

SPECIAL CASE:

DISJOINT = when events do not overlap

no outcomes in common

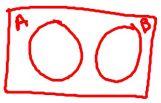
When two events are disjoint… P(A or B) = P(A U B) =

$P(A)+P(B)$

Write these rules down on your probability rules cheat sheet:

Unions:

P(A U B) = P(A) + P(B) - P(A ∩ B)



SPECIAL CASE: **Disjoint

*A and B do not overlap

* P(A ∩ B) = 0

P(A U B) = P(A) + P(B)

Try this:

1) P(K) = 0.34 and P(B) = 0.22 and P(K ∩ B) = 0.13.

(a) Are K and B disjoint? Why or why not?

No. $P(K \cap B) \neq 0$

(b) What is P(K U B)? $= P(K) + P(B) - P(K \cap B)$

$= 0.34 + 0.22 - 0.13 = 0.43$

2) P(H) = 0.57 and P(J) = 0.32 and P(J ∩ H) = 0.20.

(a) Are H and J disjoint? Why or why not?

No. $P(J \cap H) \neq 0$

(b) What is P(H U J)?

$= P(H) + P(J) - P(H \cap J)$

$= 0.57 + 0.32 - 0.20 = 0.69$

3) Mike & Ike's have 4 flavors:

Red 30%, Green 35%, Yellow 25%, Orange 10%.

(a) Are picking flavors disjoint events? Why?

Yes. $P(Y \cap O) = 0$

(b) What is the probability of picking a red or a green?

$P(R \cup G) = 65\%$

(c) What is the probability of picking a green or an orange?

$P(G \cup O) = 45\%$

Example #3: Picking poker chips from a bag ***without*** replacement

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

$P(R) = 3/10 \quad P(G) = 2/10 \quad P(B) = 5/10$

What is the probability of picking a red and then a blue?

$P(R \cap B) = (3/10) \cdot (5/9) = 1/6 = P(R) \cdot P(B|R)$

What is the probability of picking a blue and then a blue?

$P(B \cap B) = (5/10) \cdot (4/9) = 2/9 = P(B) \cdot P(B|B)$

What is the probability of picking a green and then a red?

$P(G \cap R) = (2/10)(3/9) = P(G) \cdot P(R|G)$

$\neq$

$P(G) \cdot P(R)$

VOCAB:

Conditional Probabilities =

Prob. of an event happening if another event happened first.

given that

$P(B|A) =$

1st event

INTERSECTION

So... to find the probability of event A AND event B we....

$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B|A)$

Examples #4: The probability of hitting a homerun is 30%. What is the probability of hitting 2 HRs in a row?

$P(H \cap H) = 0.09$

$P(H)$

$P(H^c)$

How about 3 HRs in a row?

$P(H \cap H \cap H) = 0.027$

How about 2 HRs then a non-homerun?

$P(H \cap H \cap H^c) = 0.063$

What is the probability that the first homerun you hit is your 4th hit?

$P(H^c \cap H^c \cap H^c \cap H) = 0.1029$

Example #5: Picking poker chips from a bag ***with*** replacement:

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

$P(R) = 3/10 \quad P(G) = 2/10 \quad P(B) = 5/10$

What is the probability of picking a red chip and a blue chip?

$P(R \cap B) = (3/10) \cdot (5/10) =$

What is the probability of picking a green chip and a red chip?

$P(G \cap R) = (2/10)(3/10)$

What's the probability of picking a blue chip and a blue chip?

$P(B \cap B) = (5/10)(5/10)$

~~What was different about these two examples?~~

Prob. of 4 Red in a row?

$P(R \cap R \cap R \cap R) = (3/10)(3/10)(3/10)(3/10)$

$P(G^c \cap G^c \cap G^c \cap G^c \cap G) =$

1st green chip is the 5th pick?

SPECIAL CASE:

INDEPENDENT =

one event happening doesn't affect the next event

When two events are independent, P(B|A) = P(B).

Write this new rule down on your probability rules cheat sheet:

INTERSECTIONS:

$P(A \cap B) = P(A) * P(B|A)$ or $P(B|A) = \frac{P(A \cap B)}{P(A)}$

SPECIAL CASE: **INDEPENDENT (A happening doesn't affect B happening)

$P(A \cap B) = P(A) * P(B)$ $P(B|A) = P(B)$

y this:

1) P(Y) = 0.35 and P(K) = 0.41 and P(Y ∩ K) = 0.13.

a. Are Y and K independent? Why or why not?

$P(Y \cap K) \neq P(Y) \cdot P(K)$ NO

0.13

b. What is P(Y|K)?

$P(Y|K) = \frac{P(Y \cap K)}{P(K)} = \frac{0.13}{0.41}$

c. What is P(K|Y)?

$P(K|Y) = \frac{P(K \cap Y)}{P(Y)} = \frac{0.13}{0.35} = 0.371$

d. Are Y and K disjoint? = 0.317

$P(Y \cap K) \neq 0$

NO

2) P(H) = 0.55 and P(D) = 0.37 and P(D|H) = 0.20.

a. Are H and D independent? Why or why not?

$P(D|H) \neq P(D)$ NO

0.20 0.37

b. What is P(D ∩ H)?

$P(D \cap H) = P(D|H) \cdot P(H) = (0.20)(0.55) = 0.11$

c. What is P(H|D)?

d. Are H and D disjoint?

$P(H \cap D) \neq 0$

NO

Example:

1) Suppose 40% of cars that Americans drive are manufactured in Japan, 30% are from Germany, 10% are from the US, and 20% are from other countries.

(a) Create a probability model

Car	Japan	Germany	US	Other
P(Car)	0.40	0.30	0.10	0.20

(b) Are cars disjoint from each other?

Yes. Cars are from one country

(c) Are cars independent from each other?

$P(J \cap G) = 0$

Yes. Cars don't affect each other

(d) If I randomly select a car, what's the probability the car is NOT from Germany?

$P(G^c) = 0.70$

(e) If I randomly select a car, what's the probability the car is from Germany or the US?

$P(G \cup US) = 0.30 + 0.10 = 0.40$

(f) If I randomly select 3 cars, what's the probability they are all from Japan?

$P(J \cap J \cap J) = (0.4)(0.4)(0.4) = 0.064$

(g) What's the probability the 4th car I select is the first one from Japan?

$P(J^c \cap J^c \cap J^c \cap J) = 0.0864$

(h) What is the probability that I select 4 cars and none are from the US?

$P(US^c \cap US^c \cap US^c \cap US^c) = 0.6561$

At a local track meet 62% of the participants were girls, 29% of the participants were from private schools, and 18% of the participants were girls from private schools. Draw a Venn Diagram and answer the questions.

(a) What is the probability that a randomly selected student was a girl or from a private school?

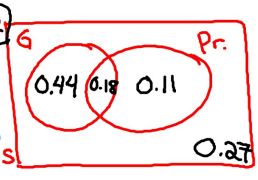
$P(G \cup Pr) = 0.73$

(b) What is the probability that randomly selected girl was from a private school?

$P(Pr|G) = \frac{P(Pr \cap G)}{P(G)} = \frac{0.18}{0.62} = 0.2903$

(c) What is the probability that a randomly selected public school student was a girl?

$P(G|Pr^c) = \frac{0.44}{0.71} = 0.6197$



(d) Is being a girl and being from a private school disjoint?

$P(G \cap Pr) \neq 0$. NO

(e) Is being a girl and being from a private school independent?

$P(B|A) = P(B)$

$P(Pr|G) = P(Pr)$

$0.2903 = 0.29$ ✓ Yes

$P(G) = 0.62$ $P(G \cap Pr) = 0.18$

$P(Pr) = 0.29$